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System Reliability – Ductility and Redundancy

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Content

- Cost E55:WG3 - objectives
- System Reliability - facts
- System Reliability – Ductility & Redundancy - facts
- Example: *System Reliability Analysis of Timber Structural System with Ductile Behavior*
- Future Work

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COST E55:WG3 – Objectives

- Characterisation of multi-scale variability in timber structures
- Analysis of system effects for several types of timber structures
- Qualification of robustness as a characteristic of timber structures
- Establishing a framework for reliability based design and assessment of timber structural systems based on these considerations.

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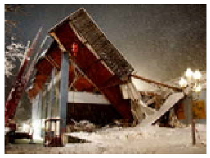
COST E55:WG3 – Objectives

- Ballerup arena, Copenhagen, Denmark — Ice skating arena, Bad Reichenhall, Germany

2 out of 12 main trusses collapsed

Total collapse



- Hazards (design error, unforeseen incidents, …): correlated / uncorrelated for different elements?
- Connection between main trusses: strong / weak ?
- Brittle / ductile failure type?

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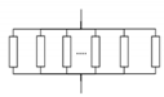
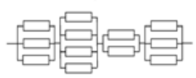
System Reliability

- Series system

- Parallel system



- Hybrid system



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System Reliability

- Element

$$P_f = P\left(M \le 0\right) = P\left(R(\overline{X}) - S(\overline{X}) \le 0\right) = P\left(g(\overline{X}) \le 0\right)$$

$$g(\overline{X}) = 1 - \left(\frac{\sum_i S_{t,i}}{(z_{d,A} \cdot R_{t,0})} + \frac{\sum_i S_{m,i}}{z_{d,M} \cdot R_m} \right) \cdot X_M = 0$$

- Series system

$$P_f^S = P\left(\bigcup_{i=1}^{m} \{M_i \le 0\}\right) = P\left(\bigcup_{i=1}^{m} \{g_i(\mathbf{X}) \le 0\}\right) = P\left(\bigcup_{i=1}^{m} \{g_i\left(\mathbf{T(U)}\right) \le 0\}\right) \approx \Phi(-\beta^s)$$

- Parallel system

$$P_f^P = P\left(\bigcap_{i=1}^{m} \{M_i \le 0\}\right) = P\left(\bigcap_{i=1}^{m} \{g_i(X) \le 0\}\right) \approx \Phi(-\beta^p)$$

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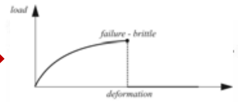
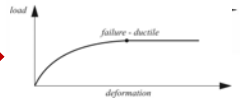
System Reliability – Ductility & Redundancy

- The behavior of structural failure modes after failure is important for assessing the safety (Robustness) of a structural system. Two extreme failure modes are:
  - brittle failure mode



  - ductile failure mode



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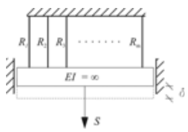
System Reliability – Ductility & Redundancy

- Simplified system modeling of ductile/brittle structural system
  - Parallel system with *m* elements
  - Perfect ductile / brittle elements
  - Load distribution after element failure

$$F_{sys} = \max_{\delta}\left(\sum_{i=1}^{m} R_i(\delta) - S \leq 0\right) = \bigcap_{\delta}\left(\sum_{i=1}^{m} R_i(\delta) - S \leq 0\right)$$



- ✓ Daniels, H.E. *The statistical theory of the strength of bundles of threads, Part I.* in *Proceedings of the Royal Society*. 1945
- ✓ Frangopol D.M. and Curley J.P., *Effects of damage and redundancy on structural reliability.* Journal of Structural Engineering, 1987. **113**(7): p. 1533-1549.
- ✓ Gollwitzer, S. and R. Rackwitz, *On the reliability of Daniels systems.* Structural Safety, 1990. **7**: p. 229-243.
- ✓ Baker, J.W., M. Schubert, and M.H. Faber, *On the assessment of robustness.* Journal of Structural Safety., 2007. **30**(3): p. 253-267.

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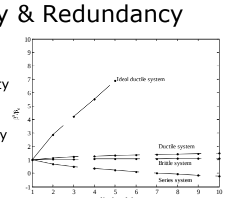
System Reliability – Ductility & Redundancy

- For a small number of elements the brittle system behaves much like the series system
- As number of elements increases the reliability of parallel system increases significantly (and vice-versa for the series system)
- As the ductility increases linearly the reliability of the system increases much steeper (exponentially), so a relatively little ductility accounts for a considerable extra reliability
- At last increases in correlation between elements imply a system reliability decrease
- In summary, if there is a moderate degree of ductility, ductile systems will provide significant extra reliability only if elements are low correlated or with no correlation at all and if the load variability is not high



- Ductile material - as number of elements increases - more robust

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System Reliability – Ductility & Redundancy

- Redundancy measures

$$RI = \frac{P_{f(dmg)} - P_{f(sys)}}{P_{f(sys)}}$$

$$\beta_R = \frac{\beta_{\text{intact}}}{\beta_{\text{intact}} - \beta_{\text{damaged}}}$$

- ✓ Frangopol D.M. and Curley J.P., *Effects of damage and redundancy on structural reliability*. Journal of Structural Engineering, 1987. **113**(7): p. 1533-1549.
- The terms redundancy, robustness and static indeterminacy are often used as synonymous.
- Structural redundancy can be defined as the ability of the system to redistribute among its members the load which can no longer be sustained by some other damaged members.
- Structural robustness can be considered as the ability of the system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself.
- Redundancy is associated with the degree of static indeterminacy. However, the degree of static indeterminacy is not a consistent measure for structural redundancy. In fact, structures with lower degrees of static indeterminacy can have a greater redundancy than structures with higher degrees of static indeterminacy.
- In general a redundant system is believed to be more robust than non-redundant systems – but this is not always the case as illustrated by the failures of the Ballerup Super Arena and the Bad Reichenhall ice arena.

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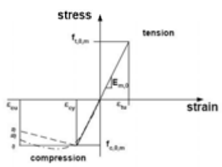
Ductile/Brittle Material Behaviour

- In general timber is considered to be a brittle material, because failure occurs suddenly, without any warning. This can be considered as an obstacle when comparing to other materials like steel. It has none or a very little ductility in the tensile area, while in compressive area linear elastic-plastic behaviour can be assumed



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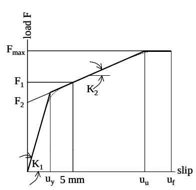
Different Ductility Definitions

I



Ductility definitions

$$D_f = \frac{u_f}{u_y} \quad (1) \qquad D_u = \frac{u_u}{u_y} \quad (2)$$

$$C_d = \frac{u_f - u_y}{u_f} \quad (3) \qquad D_{f/u} = \frac{u_f}{u_u} \quad (4)$$

$$D_{s/u} = \frac{K_1}{F_1} u_u \quad (5) \qquad D_{s/f} = \frac{K_1}{F_1} u_f \quad (6)$$

$$D_{uy} = u_u - u_y \quad (7) \qquad D_{fy} = u_f - u_y \quad (8)$$

$$D_{fu} = u_f - u_u \quad (9) \qquad E_d = \int_{u=0}^{u=u_f} f(F,u)\,du \quad (10)$$

- it can be noted that some of the definitions are 'relative' whereas some other are 'absolute'

Stehn. L and Björnfot, A. *Comparison of different ductility measurements for nailed steel-to-timber connections. Proceedings of WCTE 2002, Shah Alam*, Malasyia.

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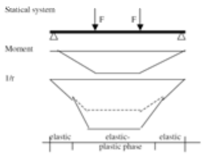
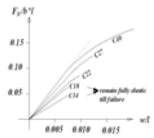
Ductile/Brittle Material Behaviour

- The better grade timbers do exhibit some ductile behavior, but the deviation from the straight line of the force-deflection-relationship is a minimum. Only when very high grade timber (C35 or C40) is used can a marked deviation from the straight line be expected.





Brunner, M. *On the Compressive Strength of Timber in Bending*. 2004.

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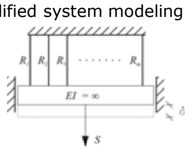
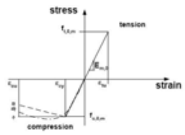
Example

- Simplified system modeling of ductile timber structural system





$$F_{sys} = \max_{\delta}\left(\sum_{i=1}^{m} R_i(\delta) - S \le 0\right) = \bigcap_{\delta}\left(\sum_{i=1}^{m} R_i(\delta) - S \le 0\right)$$

Ductility measure $D = \frac{u_f}{u_y} = 1,2,..,4$

- Parallel system with *m* elements
- Perfect ductile / brittle
- Load distribution after element failure
- Dead load (G) and live load (Q) Normal and Gumbel distributed
- Resistances (R) Lognormal distributed
- Limit state function

$S = (1-\alpha)G + \alpha Q$

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Stochastic Model – JCSS

Property	Failure type
Bending strength R_m	Ductile[1]
Tension parallel to the grain $R_{t,0}$	Brittle
Tension perpendicular to the grain $R_{t,90}$	Brittle
Compression strength par. to the grain $R_{c,0}$	Ductile
Compression strength perp. to the grain $R_{c,90}$	Ductile with reserve
Shear R_v	Brittle

Property	Distribution	COV
Bending strength R_m	Lognormal	0.25
Bending MOE E_m	Lognormal	0.13
Density ρ	Normal	0.1

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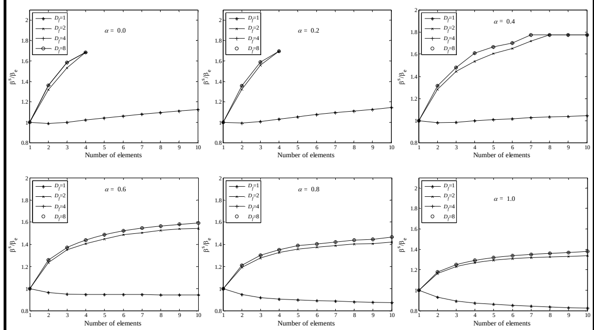
Results



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Future Work (on-going research in COST E55)

- The ductility and redundancy effects for improving the robustness of timber structures are considered for different types of structures
  - Different extraordinary exposure events are considered
  - Damages to structural elements are defined
  - Depending on the degree of ductility a damaged element maintains/loses load carrying
  - Robustness/Redundancy indices calculated

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